Upper Bounds on Massive Lepton Pair Production,

$pN \rightarrow \ell^+\ell^- X$, in the Quark-Parton Model

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ABSTRACT

We present rigorous bounds on the process $pN \rightarrow \ell^+\ell^- X$ derived

in the context of quark-parton models. As constraints we use the

positivity of the parton probability functions and data on deep inelastic

leptoproduction. Generally, our upper bounds fall below the Brookhaven

data on muon pair production. This result forces us to conclude that

the BNL data does not represent the scaling limit for this process,

and/or that the usual ideas of the (colored) quark parton model are not

all correct.

The intriguing successes of parton models for processes involving large space-like momentum transfers, especially deep inelastic lepton scattering, have been well demonstrated over the past several years. In stark contrast are the results on e^+e^- annihilation from CEA, and more recently from SPEAR. These results are in gross disagreement with parton model expectations. Another process which has been the subject of much theoretical and experimental study is the production of massive lepton pairs in hadronic collisions. This process, first discussed in the parton model by Drell and Yan,[1] is supposed to occur when a parton from one incident hadron annihilates with an antiparton from the other incident hadron, producing a heavy photon which finally decays into a lepton pair. The cross section for this process in proton-proton collisions is given by

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{4}{3} \pi \alpha^2 \tau \int_\tau^1 \frac{dx}{x} \sum_i e_i^2 [q_i(x)\bar{q}_i(\tau/x) + \bar{q}_i(x)q_i(\tau/x)] \quad (1)$$

where $\tau = Q^2/s$, e_i is the charge of a parton of type i, and $q_i(x)(\bar{q}_i(x))$ is the probability of finding a parton (antiparton) of type i with a fraction x, of the parent proton's longitudinal momentum.

The rates for this process as measured at Brookhaven by Christensen et al.,[2] have generally been larger than theoretical estimates, but no firm conclusions about whether the parton model could describe these results have been drawn. In this Letter we present some rigorous

bounds on this cross section and compare them with the BNL data.
The bounds are based on the positivity of the parton probability distri-
butions and the constraints on these distributions imposed first by the
SLAC electroproduction data[3] and later, by the neutrino experiments at
Gargamelle.[4] The probability distributions are regarded as generalized
coordinates in a variational problem, and constraints are imposed by
using the method of Lagrange multipliers generalized to incorporate
inequality constraints.[5] The positivity requirements are

$$q_i(x), \quad \bar{q}_i(x) \geq 0 \tag{2}$$

for all x and i.

In the first problem we consider we use only the data from SLAC on
νW_2 for protons and neutrons,[3] This data is quite good, and so our
bound will provide a very clean test of the parton model. In the parton
model these structure functions are given by certain linear combinations
of the parton probability distributions. Specifically, if the partons are
quarks and carry, in addition, SU(3) color, we have

$$\frac{\nu W_2^{\gamma p}(x)}{x} = \frac{4}{9} U(x) + \frac{1}{9} D(x) + \frac{1}{9} S(x)$$

$$\frac{\nu W_2^{\gamma n}(x)}{x} = \frac{4}{9} D(x) + \frac{1}{9} U(x) + \frac{1}{9} S(x)$$

where $U(x) = u_r(x) + \bar{u}_r(x) + u_w(x) + \bar{u}_w(x) + u_b(x) + \bar{u}_b(x)$, and

similarly for $S(x)$ and $D(x)$. The subscripts refer to the color degrees of freedom.

When combined with the inequalities (2), these constraints provide rather severe restrictions on the cross section (1). The full derivation of this bound is too long to describe here, and will be presented in detail (along with a number of additional results) elsewhere.[6] The result is shown and compared with the BNL data in Fig. 1. Curve A is the bound computed with the constraints described for the process $pU \rightarrow \mu^+\mu^- X$, since the target used was uranium. That is, it is a weighted average of the upper bounds for $pp \rightarrow \mu^+\mu^- X$ and $pn \rightarrow \mu^+\mu^- X$. Curve B is the same as curve A, corrected to incorporate the detection efficiency of the BNL experiment, and curve C is the 29.5 GeV/c data from Brookhaven.[7] For $0.2 \lesssim Q^2/s \lesssim 0.5$ the upper bound falls significantly below the data proving that the colored quark parton model cannot possibly describe the result.

Corresponding bounds for the quark parton model without color are obtained by multiplying curves A and B by a factor of three. Even in this theory, the data comes dangerously close to violating the bounds.

In the problem considered above, the q_i and $\bar{q}_i$ distributions enter symmetrically. However, for nonzero x, the usual notions of the parton model suggest that there is a significant difference between the quark and antiquark distributions. To implement this difference in our variational problem, we turn to a consideration of neutrino scattering. Using

the preliminary data on neutrino-nucleon scattering from Gargamelle,[4]

and the SLAC data on νW_2 for protons and neutrons, one can deduce the

four different linear combinations of quark probability distributions

shown in Fig. 2a. In the colored quark parton model, the labels of the

curves in this figure should be understood as sums over color, e.g.,

$u(x) = u_r'(x) + u_w(x) + u_b(x)$, etc. There are many reasons to be sceptical

about the preliminary data from Gargamelle, and its interpretation in

terms of parton distributions (for instance, most of the data is not at

large enough Q^2 to expect scaling), but at the very least, it is probably

indicative of what cleaner data will show. Interpolating these points and

modifying slightly the curves at small Q^2 (i.e., x)[8] we take as our "data"

the curves of Fig. 2b. A detailed discussion of the Gargamelle data and

further elaboration of Fig. 2b may be found in Ref. 6.

We can now use this data, which distinguishes between quarks and

antiquarks as constraints in a new variational problem. Again we impose

the positivity requirements (2), and maximize the functional. The result

of this tedious, but straightforward, calculation[6] is shown in Fig. 3,

where it is compared with the BNL data. Again, the upper bound falls

far below the data leading to the conclusion that the parton model and the

BNL data are incompatible. As before, the same bound derived in the

uncolored quark model is precisely three times as large as the bound

of Fig. 3.

One might worry that, since the Gargamelle data has such large

error bars, and since the process in question is so sensitive to the

size of the antiquark distributions at large x, that the bound of Fig. 3

could change drastically within the error bars of the data. However,

model studies[6] indicate that while the upper bound does change significantly

when one changes the data, it is still at least one or two orders of

magnitude smaller than the data for $0.2 \lesssim \tau \lesssim 0.6$. In any event, no

matter what the neutrino data eventually turns out to be, the bound can

never rise about that of Fig. 1 which depends only on electron scattering

and makes no assumptions about the weak interactions. We also want to

stress that the method of solution for this problem, which is given in

Ref. 6, does not depend on the form of the data in Fig. 2b and can

easily be applied to other data when it becomes available.

These bounds, which are perfectly rigorous in the context of the

usual parton models, force us to conclude that at least one of the following

statements is correct.

i) The BNL data for the experimentally observed cross section per

nucleon is not the scaling limit for the process $pN \rightarrow \mu^+ \mu^- X$.

ii) The Drell-Yan formula for $pp \rightarrow \mu^+ \mu^- X$ must be modified.

iii) The colored quark parton model is wrong.

Let us briefly comment on each possibility:

i) Nuclear effects which were thought to be well understood may,

in fact, significantly alter the observed cross section per nucleon.

Inasmuch as experiments on hydrogen will not be available for quite
some time, an experiment utilizing different nuclear targets is urgently
needed. Another effect which may be important at large τ is the inter-
actions of secondary mesons produced in the target. Since mesons are
supposed to have many more antiquarks at large x than baryons, this
effect may be important in a thick target such as that at BNL. It must be
remarked, however, that model-dependent estimates suggest that this
contribution is still at least an order of magnitude below the observed
signal.[10] Finally, it is possible that the Brookhaven experiment is not the
scaling limit for this reaction. Similar experiments at other energies
such as those being planned and carried out at NAL are obviously of
great interest.

ii) Theoretically, the most unsatisfactory aspect of the Drell-Yan
formula is its apparent neglect of strong interactions. Such interactions
are generally required in deep inelastic phenomena, especially in a
quark parton model, to dispose of the isolated quark quantum numbers.
However, these effects may leave the parton model results for deep
inelastic lepton scattering unchanged, but alter the predicted cross
sections for processes, such as the one discussed here, which involve
more than one hadron in both the initial and final states.[11] Our results
make this fundamental and unresolved question that much more salient.

iii) The colored quark parton model may be incorrect in one or both
of two ways. One the one hand, the correct parton model may have

constituents other than colored quarks. For such theories bounds

similar to those presented here can be derived. In general, we can

anticipate that the greater the mean-squared charge, the smaller the

bound will be, so the situation will be worse in most such models. On

the other hand, the ideas of the naive parton model may not be applicable

in their present form to processes involving time-like photons. (This

possibility is related to option (ii), discussed above.) Indeed, it is

intriguing that in the two large Q^2 processes involving time-like photons

which have been measured--e^+e^- annihilation and massive muon-pair

production--the observed cross sections are larger than the predictions

of the parton model.

The ideas of the parton model, which have worked so well in describing

processes with large space-like momentum transfers,seem to fail, in

their present form, in the time-like region. We have shown in this

paper, by presenting rigorous bounds in the context of parton models,

that the most popular versions cannot describe the BNL data on

$pN \to \mu^+\mu^- X$. The CEA and SPEAR results on e^+e^- annihilation, and

the present bounds on massive lepton pair production make it extremely

important to carry out further experiments in the deep time-like region,

and to focus our theoretical attention on the exciting paradoxes presented

by the parton model.

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[7] In presenting the data, we have drawn a smooth curve through the data
points and have not indicated statistical errors which range between 5%
and 25% for $0.02 \leq \tau \leq 0.45$ and are larger for larger τ. However, the
dominant errors are systematic ranging between 25% and 65% for
$0.02 \leq \tau \leq 0.61$ and are correlated from point to point. Consideration
of these errors do not vitiate our conclusions.

We also point out that the difference between calculating the average
of the upper bounds on p and n targets and the upper bounds on the average
nucleon in uranium is negligible for our purposes. For further details,
see Ref. 6.

[8] The modification is desirable for several reasons; for example, to make the probability distributions nonpositive.

[9] Actually, the curve in Fig. 3 is a bound for the process $pp \rightarrow \mu^+ \mu^- X$ not $pU \rightarrow \mu^+ \mu^- X$ as in Fig. 1. However, the bounds for proton and neutron targets typically differ by a factor of two or so, and for the purpose of Fig. 3 this difference is insignificant. See Ref. 6 for more details.

[10] G. Farrar and G. C. Fox, private communication.

[11] Different points of view on this question are suggested by R. P. Feynman, Photon-Hadron Interactions, W. A. Benjamin, Inc. (1972), p. 152; and P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B33 (1971), 221. See also Ref. 6 above.

FIGURE CAPTIONS

Fig. 1   (A) Upper bounds for the process $pU \rightarrow \mu^+ \mu^- X$ calculated as a weighted average of bounds on $pp \rightarrow \mu^+ \mu^- X$ and $pn \rightarrow \mu^+ \mu^- X$. (B) Same bound corrected to include detection efficiency at BNL. (C) 29.5 GeV/c data from Brookhaven.

Fig. 2   (a) Parton distribution functions gleaned from SLAC and Gargamelle data.

(b) Modified version of Fig. 2a (see text).

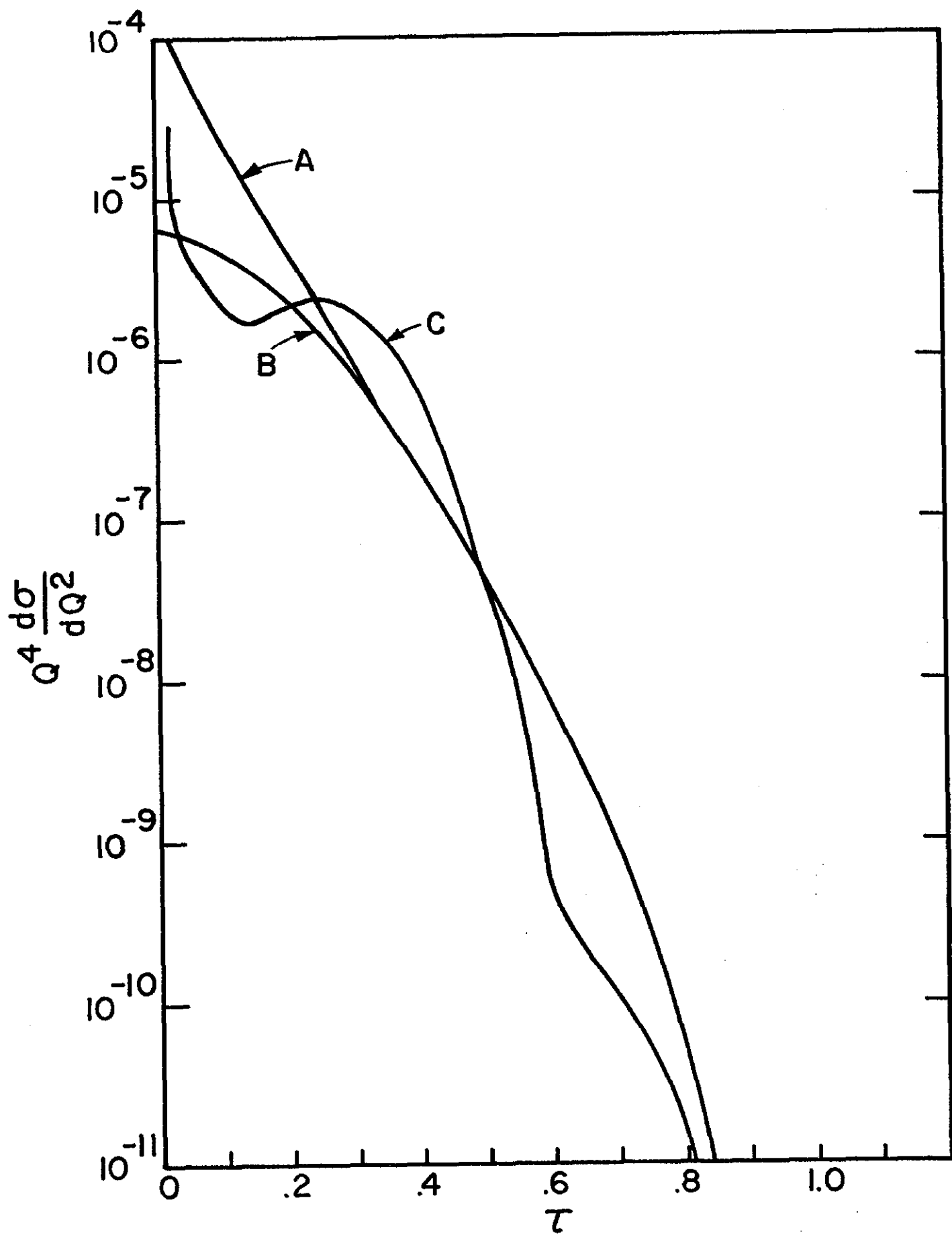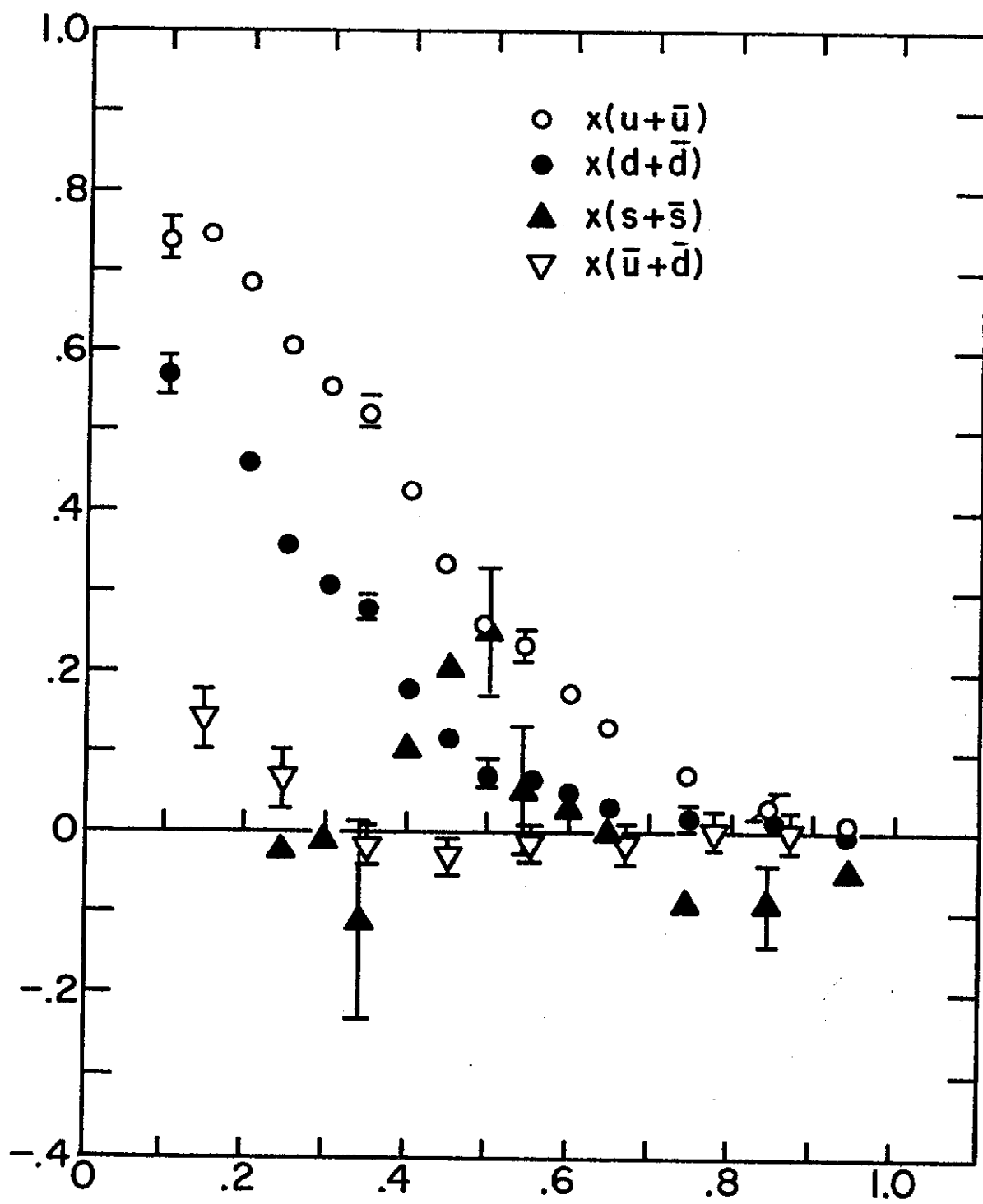Fig. 3   Upper bound derived using data of Fig. 2 (solid line) compared with BNL data (dashed line).
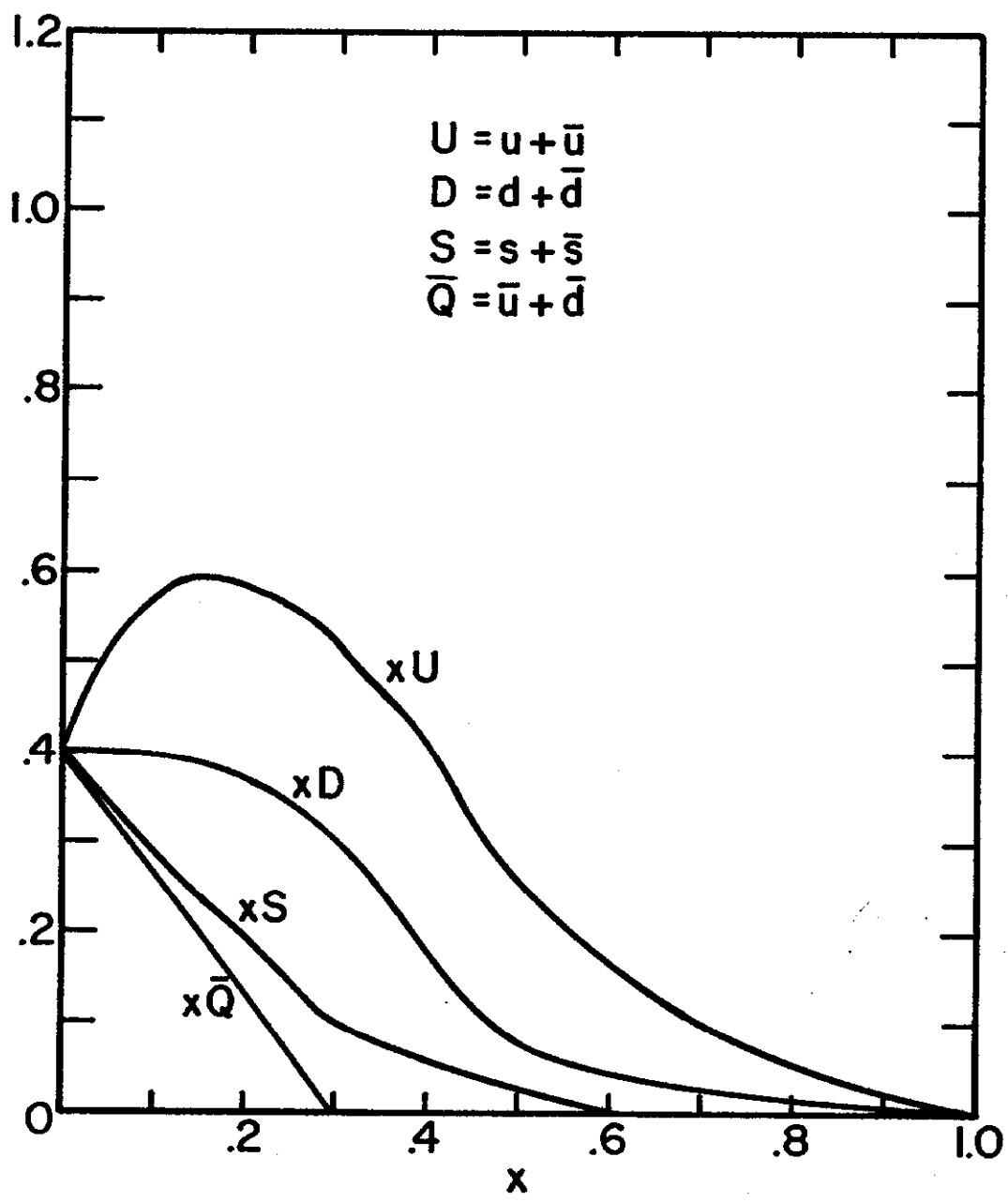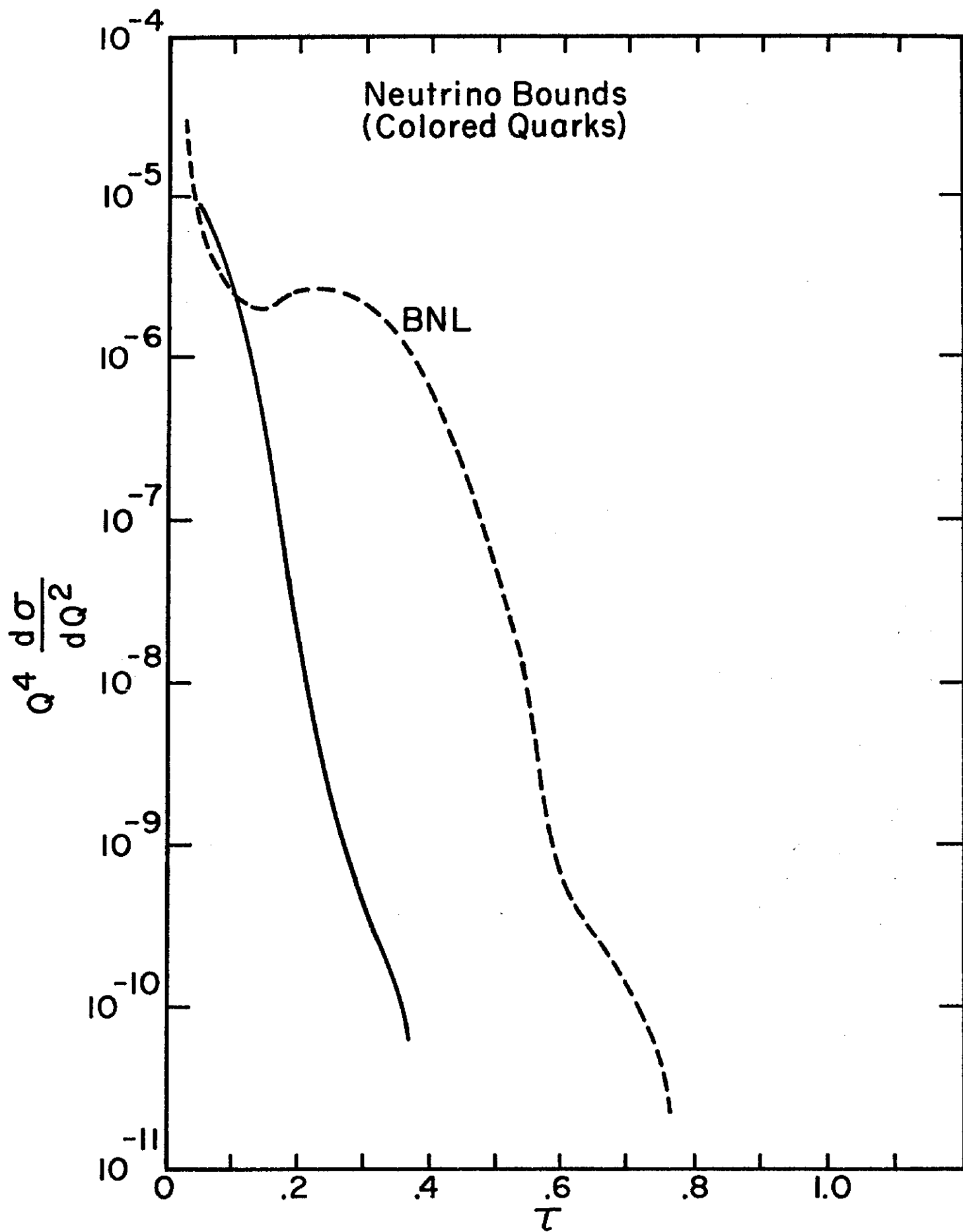
Figure 1

Figure 2a

Figure 2b

Figure 3